Smith **Math 290** **Fall 2008**

Exam 2 **October** 16, 2008

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do both of these "Computational" problems

C.1. [8, 7 Points] Given the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 1 & 0 & 3 & 2 \\ 2 & 6 & 1 & 8 & -3 & 6 & 4 \\ 1 & 3 & 4 & -3 & -2 & -1 & 2 \\ 2 & 6 & -3 & 16 & 7 & 20 & 4 \\ 1 & 3 & 0 & 5 & 2 & 7 & 2 \end{bmatrix}$.

1. (a) Write down the matrices C, O, K, L that are found in the extended reduced row-echelon form of A.

   (b) Compute the column space of A by writing the null space of L, $N(L)$ as the span of a linearly independent set of vectors.

C.2. [15 points] Consider the sets $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ -3 \\ 0 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} 1 \\ 0 \\ \frac{7}{3} \\ -\frac{8}{3} \\ -\frac{1}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -\frac{2}{3} \\ \frac{7}{3} \\ \frac{2}{3} \end{bmatrix} \right\}$.

1. (a) Prove that $\langle S \rangle \subseteq \langle T \rangle$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] There is a theorem in our text proving that if A is an invertible matrix, then $\left(\overline{A}\right)^{-1} = \overline{(A^{-1})}$. Prove this theorem.

M.2. [15 Points] Prove that if A in an $m \times n$ matrix and B is $n \times p$ then the column space of AB is contained in the column space of A. That is, prove $C(AB) \subseteq C(A)$.

M.3. [15 Points] Prove that if a set S is linearly dependent and S is a subset of the set T, then T is also linearly dependent.

Induction Problem

1. [12, 3 Points] Suppose A and B are square matrices of equal size and we know that $AB = BA$.

   (a) Use mathematical induction to prove that $A^n B = BA^n$ for every positive integer n.

   (b) Use your result from part (a.) above to prove that $A^n B^m = B^m A^n$ for every pair of positive integers n, m.

Do one (1) of these "Other"

T.1. [15 Points] Prove that if $S = \{\vec{u}_1, \cdots, \vec{u}_p\}$ is a linearly independent set of vectors and $\vec{v} \notin \langle S \rangle$ then the set $T = \{\vec{u}_1, \cdots, \vec{u}_p, \vec{v}\}$ is also linearly independent.

T.2. [15 Points] Prove that if $T = \left\{\vec{t}_1, \cdots, \vec{t}_n\right\}$ is a set and $S = \{\vec{u}_1, \vec{u}_2\}$ is another set contained in the span of T, (that is, $S \subseteq \langle T \rangle$) then $\langle S \rangle \subseteq \langle T \rangle$ .More specifically, show that if $\vec{u}_1 \in \langle T \rangle$ and $\vec{u}_2 \in \langle T \rangle$, then any linear combination of $\vec{u}_1$ and $\vec{u}_2$ is also in $\langle T \rangle$.